SEA. Jour. Math. & Math.Sc.
Vol.1 No1 (2002), pp.71-82

THE ASKEY-WILSON OPERATOR AND THE ${}_6\Phi_5$ SUMMATION FORMULA

Shaun Cooper
Institute of Information and Mathematical Sciences, Massey University-Albany,
Private Bag 102904, North Shore Mail Centre
Auckland, New Zealand
e-mail : s.cooper@massey.ac.nz

(Received on November, 25, 2002)

ABSTRACT : The summation formula $\sum_{k=0}^{n} \frac{(-n)_k (a)_k}{k!(c)_k} = \frac{(c-a)_n}{(c)_n}$ can be proved by expanding each term in the identity $(1-x)^{-\alpha}(1-x)^{-\beta}=(1-x)^{-\alpha-\beta}$ by the binomial theorem, equating coefficients of x^n on both sides and relabelling parameters. The aim of this paper is to use the Askey-Wilson operator D_q and its index lad $D_q^{\alpha} D_q^{\beta} = D_q^{\alpha+\beta}$ to give a similar proof of the summation formula for a terminating, very well poised ${}_6\phi_5$ series.

AMS subject Classification: Primary 33D20, 22C20; Secondary 39A70, 47B39.

1. INTRODUCTION

The Wilson and Askey-Wilson divided difference operators arise naturally in the theory of the Wilson and Askey-Wilson polynomials, respectively; see [2, pp. 32-36]. Properties and applications of these operators have since been studied by several authors. Askey [1] used the Askey-Wilson operator to give a simple proof of Rogers' connection coefficient formula for the continuous q-ultraspherical polynomials. The Askey-Wilson operator and another operator were used by Kalnins and Miller [7] to derive the orthogonality of the Askey-Wilson polynomials. Ismail [6] gave new proof